

On the Microfoundation of Linear Oligopoly Demand

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On the Microfoundation of Linear Oligopoly Demand

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Abstract

We critically assess the representative consumer model that forms the foundation of a well-known class of linear oligopoly demand structures. It is argued that this approach has several limitations. We present an alternative microeconomic foundation by deriving the same demand system directly from a population of heterogeneous buyers. Our approach can be easily adapted to different demand specifications.

Keywords: Microfoundations; Oligopoly Theory; Product Differentiation; Representative Consumer Models.

JEL Codes: B4; L1.

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1 Introduction

A well-known way of describing the buyers' side of an oligopoly market is through a linear horizontally differentiated demand model. For the case of duopoly, the (direct) demand structure generally takes the following form:

$$x_1(p_1, p_2) = a_1 - b_1 \cdot p_1 + c \cdot p_2,$$

$$x_2(p_1, p_2) = a_2 - b_2 \cdot p_2 + c \cdot p_1,$$

where price and quantity are positive and respectively given by p_i and x_i , for $i = 1, 2$.¹ It is, moreover, commonly assumed that $a_i, b_i, c > 0$ and $b_i > c$, for $i = 1, 2$, so that a firm's demand depends negatively on its own price, positively on the rival's price and own effects dominate cross effects. This demand system can be roughly interpreted as follows. If, say, firm 1 raises its price slightly, then *ceteris paribus* some of its customers walk away and either go home or visit firm 2 instead. Likewise, lowering price attracts additional buyers, some of whom switch from the competing firm.

The traditional foundation for this demand specification does not come from a group of heterogeneous buyers, however, but from a representative consumer who on behalf of an unspecified buyer population maximizes a quadratic aggregate welfare function. The most popular variations of this type are due to Bowley (1924) and Shubik and Levitan (1980).²

In the field of macroeconomics, such a representative agent approach has been heavily criticized by many. One reason for this is that transforming individual preferences into representative aggregate preferences often proves problematic. It is, for instance, quite possible that the representative agent prefers A to B, whereas each and every represented buyer prefers B to A.³ For this and other reasons, many macroeconomists are reluctant to take this approach and some even went as far as to effectively compose a requiem for the representative consumer.⁴ This is in stark contrast to the fields of microeconomics and industrial organization, where the use of such a fictitious agent is widely accepted. What makes this particularly

¹See, for instance, Singh and Vives (1984). This setting can be easily generalized to an n -firm variant. See, for example, Häckner (2000).

²See Martin (2002) for a detailed discussion of both these models.

³This Pareto inconsistency has been clearly established by Jerison (1984). See also Dow and da Costa Werlang (1988).

⁴See, for example, Kirman (1992).

surprising is that a rationale for this approach is commonly missing.⁵

In this paper, we have two main goals. The first is to provide a critical assessment of the representative consumer as a foundation for the above linear demand structure. Specifically, we argue that it is both *inaccurate* and *inadequate*. It is inaccurate as the representative agent's aggregate utility function has no (clear) connection with the objectives of those represented.⁶ It is inadequate because a justification for both the utility specification and the solution approach is missing. Taken together, this leads us to conclude that the popular linear oligopoly demand structure lacks a proper foundation.

We then proceed with our second goal, which is to argue that quadratic representative consumer models are effectively redundant. We do so by showing that the same demand structure can be easily derived directly from a population of heterogeneous consumers. A main advantage of this approach is that it is explicitly based on simple buyer behavior at the micro-level and therefore has a natural interpretation.

A couple of recent papers have raised some red flags regarding the use of a representative agent with a quadratic utility function. Kopel, Ressi and Lambertini (2017), for instance, show that seemingly similar quadratic aggregate utility functions may give rise to fundamentally different demand systems. In turn, this might lead to radically different policy implications. Amir, Erickson and Jin (2017) provide a thorough study of several characteristics of the quadratic utility specification. Among other things, they establish that strict concavity of the utility function is a necessary condition for the corresponding demand system to be well-defined.

The remainder of this paper is organized as follows. The next section discusses the quadratic representative consumer utility function and highlights several problematic features of this specification. Section 3 presents a microfoundation for linear oligopoly demand. Section 4 concludes.

⁵As a telling example, both Bowley (1924) and Shubik and Levitan (1980) introduce this approach as an illustration and do not provide an explanation or justification for the specification of their representative consumer's utility function.

⁶The exception here would be when the population of represented buyers is assumed to all possess the same utility function. Yet, in that particular case it is not clear what exactly would be the added value of a representative agent.

2 Quadratic Representative Consumer Models

In this section, we express some concerns about the above mentioned class of quadratic representative consumer models. Specifically, we raise several issues regarding the shape of the objective function, the derivation of the corresponding demand functions and the relation between the products involved.

2.1 Issue 1: the Objective

Both the Bowley (1924) and the Shubik-Levitan (1980) demand specifications are derived from a representative consumer *gross* utility function that takes the following general form:

$$U(x_1, x_2) = \alpha \cdot (x_1 + x_2) - \beta \cdot (x_1 + x_2)^2 - \gamma \cdot (x_2 - x_1)^2,$$

with $\alpha, \beta, \gamma > 0$.⁷ Notice that the way in which we present the objective function, it effectively consists of three distinct parts. Starting with the third, $\gamma \cdot (x_2 - x_1)^2$, this part captures the complementarity between both products. Consistent with classic consumer theory, utility is *ceteris paribus* higher with a more balanced consumption plan. In fact, the representative agent is induced to buy both products in equal amounts ($x_1 = x_2$) so that the third-term disutility is minimized and *de facto* disappears.

The first two parts, $\alpha \cdot (x_1 + x_2) - \beta \cdot (x_1 + x_2)^2$, capture utility coming from total rather than relative consumption and express the extent to which the goods are considered substitutes. These components indicate that the consumer derives utility from consuming more products, but only up to a certain amount. That is, utility increases at low levels of total consumption through the first term, but at higher levels of total consumption the second term starts to dominate the first. This implies that there is a point at which the consumer is satiated. Notice that this holds even when the representative consumer would not face a budget constraint. Contrary to the third term favoring balance in consumption, this therefore is at odds with traditional consumer theory.⁸ Indeed, the fact that the objective function has a unique maximum makes that the common assumption of nonsatiation is violated.

⁷See Bowley (1924, p.56) and Shubik and Levitan (1980, p.69).

⁸See, for instance, Chapter 1 of Jehle and Reny (2001) for a detailed discussion of classic consumer theory.

2.2 Issue 2: the Solution

To solve the representative consumer problem, one naturally needs to take account of the cost of consumption. In the following, we point out that the linear oligopoly demand structure will only result from the representative agent's maximization problem under fairly specific, and arguably strong, assumptions.

Towards that end, let F be a set of vectors $(x, m) \in \mathbb{R}^n \times \mathbb{R}$ and consider the utility function $V : F \mapsto \mathbb{R}$. For $(x, m) \in F$, therefore, utility $V(x, m)$ is obtained from consuming an amount of $x \in \mathbb{R}^n$ goods as well as from the unspent money m . Moreover, let the vector of prices be given by $p \in \mathbb{R}^n$. If the available income is $I \geq 0$, then the representative consumer faces the following general maximization problem:

$$\begin{aligned} \max_{x \in F} \quad & V(x, m) \\ \text{s.t.} \quad & m = I - p \cdot x, \\ & m \geq 0, \end{aligned}$$

where $p \cdot x$ is total expenditure.

It can be easily verified, however, that the linear oligopoly demand system is the solution to:

$$\begin{aligned} \max_{x \in F} \quad & U(x) + m \\ \text{s.t.} \quad & m = I - p \cdot x. \end{aligned}$$

Thus, in light of the general maximization problem, $V(x, m) = U(x) + m$ and the constraint $m \geq 0$ is ignored. Observe that this specification effectively treats expenditures as a disutility, which is linearly subtracted from the gross utility function. At first sight this may seem natural and innocuous, but it does imply two strong assumptions:

[A1] The representative consumer's utility function is quasi-linear in money;

[A2] The representative consumer can borrow unlimited amounts of money for free.

The first condition means the absence of a *wealth effect* as utility is linearly increasing in money. Irrespective of whether utility is high or low, the marginal gain of an extra dollar is the same. The second states that the budget restriction has no bite and is effectively non-existent. Indeed, this approach implies that the budget constraint is never binding even when the representative consumer would have no income at all.⁹

⁹Amir, Erickson and Jin (2017) point out that the representative consumer's budget should be sufficiently high for an interior solution in which income effects are absent, which is an alternative interpretation of A2.

2.3 Issue 3: the Products

Following the textbook approach and thus assuming A1 and A2, the representative consumer picks x_1 and x_2 to maximize:

$$V(x_1, x_2) = \alpha \cdot (x_1 + x_2) - \beta \cdot (x_1 + x_2)^2 - \gamma \cdot (x_2 - x_1)^2 + I - p_1 \cdot x_1 - p_2 \cdot x_2,$$

which gives

$$\begin{aligned} x_1 &= \frac{\alpha}{4\beta} - \left(\frac{1}{8\beta} + \frac{1}{8\gamma}\right) \cdot p_1 + \left(\frac{1}{8\gamma} - \frac{1}{8\beta}\right) \cdot p_2, \\ x_2 &= \frac{\alpha}{4\beta} - \left(\frac{1}{8\beta} + \frac{1}{8\gamma}\right) \cdot p_2 + \left(\frac{1}{8\gamma} - \frac{1}{8\beta}\right) \cdot p_1. \end{aligned}$$

Goods are substitutes in *price* when $x_1(p_1, p_2)$ is increasing in p_2 and $x_2(p_1, p_2)$ is increasing in p_1 . Notice that this is the case precisely when $\frac{1}{8\gamma} > \frac{1}{8\beta}$, which is equivalent to $\gamma < \beta$.

Goods are substitutes in *utility* when for every $\eta_1 > 0$ there is an $\eta_2 > 0$ such that:

$$U(x_1 - \eta_1, x_2 + \eta_2) = U(x_1, x_2).$$

That is, every decrease in utility resulting from a reduction in the consumption of good 1 can be compensated by an increase in the consumption of good 2.¹⁰ If we consider x_2 an implicit function $X_2(x_1)$, then $X_2' < 0$. Implicit differentiation yields:

$$\alpha \cdot (1 + X_2') - 2\beta \cdot (x_1 + X_2) \cdot (1 + X_2') - 2\gamma \cdot (x_1 - X_2) \cdot (1 - X_2') = 0.$$

Rearranging gives:

$$X_2'(x_1) = \frac{2\gamma \cdot (x_1 - x_2) + 2\beta \cdot (x_1 + x_2) - \alpha}{2\gamma \cdot (x_1 - x_2) - 2\beta \cdot (x_1 + x_2) + \alpha}.$$

Thus, $X_2' < 0$ requires

$$2\gamma \cdot (x_1 - x_2) + 2\beta \cdot (x_1 + x_2) - \alpha < 0 \implies (\beta + \gamma) \cdot x_1 + (\beta - \gamma) \cdot x_2 < \frac{\alpha}{2}$$

and

$$2\gamma \cdot (x_1 - x_2) - 2\beta \cdot (x_1 + x_2) + \alpha > 0 \implies (\beta + \gamma) \cdot x_2 + (\beta - \gamma) \cdot x_1 < \frac{\alpha}{2}$$

or the reverse.

¹⁰It is noteworthy that there is an alternative interpretation of substitution in *utility* in the literature that dates back as far as Edgeworth (1881). In that case, two goods, x and y , are considered substitutes in utility when $\partial^2 U / \partial x \partial y < 0$ and $\partial^2 U / \partial y \partial x < 0$. It can be easily verified that when both goods are substitutes in utility according to this definition, then they are indeed also substitutes in price and *vice versa*.

Note that the above inequalities do not hold for all consumption bundles (x_1, x_2) . For instance, $x_1 = 0$, $x_2 = 1$ and $\beta - \gamma < \frac{\alpha}{2} < \beta + \gamma$ violates the inequalities. In this model, therefore, products are never pure substitutes or complements. In particular, there is always a combination (x_1, x_2) for which x_1 and x_2 are complements in utility and substitutes in price.¹¹

This is a remarkable result in that one would expect a clear connection between the properties of the utility function and the properties of the corresponding demand structure. This limited explanatory power is particularly problematic since the representative consumer's utility function is ultimately intended to serve as a microfoundation for linear oligopoly demand.

3 A Microeconomic foundation for Linear Oligopoly Demand

In the previous section, we have highlighted some problematic features of quadratic representative consumer models. We now proceed by presenting an alternative microfoundation for the linear oligopoly demand structure as described above:

$$x_1(p_1, p_2) = a_1 - b_1 \cdot p_1 + c \cdot p_2,$$

$$x_2(p_1, p_2) = a_2 - b_2 \cdot p_2 + c \cdot p_1.$$

Specifically, we will show in the following how this demand system can be derived directly from a population of heterogeneous consumers.

To begin, consider a price-setting duopoly where both firms are located on the boundary of an interval $[0, 2]$. In particular, and without loss of generality, firm 1 and firm 2 are respectively situated at 0 and 2. There are three types of consumers, each with uniform population density λ_i on $[0, 2]$, $i = 1, 2, 3$. The total number of type i buyers is thus given by $2 \cdot \lambda_i$. Type 1 customers are assumed to obtain positive gross utility when buying from firm 1, $s > 0$, and no utility when buying from firm 2. By contrast, Type 2 customers attach no value to the products of firm 1 and derive positive gross utility from buying at firm 2, $v > 0$. Finally, Type 3 customers value both equally and have a willingness to pay of 4 for each.¹² Consumers buy

¹¹Indeed, it can be easily verified that the level curves are ellipses. To illustrate, let $\alpha = 4$ and $\beta = \gamma$ be (approximately) equal to 1 so that $U = 2 - (x_1 - 1)^2 - (x_2 - 1)^2$. At $U = 1$, the indifference curve is therefore a circle with center $(x_1, x_2) = (1, 1)$ and radius 1. The goods are then complements at $(x_1, x_2) = (1 + \frac{1}{2}\sqrt{2}, 1 - \frac{1}{2}\sqrt{2})$, for example.

¹²As an illustrative interpretation, one may view both firms as competing ice cream vendors where firm

no more than one unit of the product and are characterized by their location. In the spirit of spatial IO settings, there are costs associated with distance between buyer and seller and these are assumed to be linearly increasing.

Let us now specify the utility function of a Type 1 consumer located at $z \in [0, 2]$. This customer has basically three options: (1) buy from firm 1 (value $s - z - p_1$), (2) buy from firm 2 (value $z - 2 - p_2$) or (3) buy nothing (value 0). Notice that the third choice dominates the second, because $z - 2 - p_2 \leq 0$ for positive prices. Thus, the utility function of a Type 1 buyer located at $z \in [0, 2]$ effectively is

$$u_1(p_1, p_2, z) = \max\{s - z - p_1, 0\}.$$

The Type 1 customer who is indifferent between option (1) and option (3) is located at $z = s - p_1$.

The utility function of a Type 2 customer located at $z \in [0, 2]$ can be determined in a similar fashion and is given by

$$u_2(p_1, p_2, z) = \max\{v - 2 + z - p_2, 0\}.$$

The Type 2 customer who is indifferent between buying and not buying is thus located at $z = p_2 - v + 2$. Finally, the utility function of a Type 3 customer at $z \in [0, 2]$ is

$$u_3(p_1, p_2, z) = \max\{4 - z - p_1, 2 + z - p_2, 0\}.$$

Under the assumption that prices are sufficiently low, the indifferent Type 3 buyer is located at $z = 1 + \frac{1}{2}(p_2 - p_1)$.¹³

On the basis of these utility specifications, we can now derive the corresponding demand functions. For a given combination of prices (p_1, p_2) in the relevant range, demand for firm 1 is given by the sum of consuming Type 1 buyers and the part of Type 3 buyers preferring the product of firm 1.¹⁴

$$x_1(p_1, p_2) = \lambda_1 \cdot (s - p_1) + \lambda_3 \cdot \left(1 + \frac{1}{2}(p_2 - p_1)\right)$$

1 sells strawberry flavor and firm 2 sells vanilla ice. Type 1 buyers are then those customers who only like strawberry ice, for example, whereas Type 2 buyers exclusively prefer vanilla. Type 3 customers consider both and let their buying decision depend on the prices set.

¹³A sufficient condition to ensure that all Type 3 consumers buy a product is $p_1, p_2 \leq 2$.

¹⁴Indifferent Type 1 and Type 2 customers are located in the interval $[0, 2]$ when $s - 2 \leq p_1 \leq s$ and $v - 2 \leq p_2 \leq v$. Type 3 customers prefer to buy a product when $p_1 + p_2 \leq 6$.

$$= s \cdot \lambda_1 + \lambda_3 - (\lambda_1 + \frac{1}{2}\lambda_3) \cdot p_1 + \frac{1}{2}\lambda_3 \cdot p_2.$$

Demand for the products of firm 2 can be derived in a similar way.

$$x_2(p_1, p_2) = v \cdot \lambda_2 + \lambda_3 - (\lambda_2 + \frac{1}{2}\lambda_3) \cdot p_2 + \frac{1}{2}\lambda_3 \cdot p_1.$$

The above approach therefore allows one to derive any linear oligopoly demand structure of the form:

$$x_1(p_1, p_2) = a_1 - b_1 \cdot p_1 + c \cdot p_2,$$

$$x_2(p_1, p_2) = a_2 - b_2 \cdot p_2 + c \cdot p_1,$$

where $\lambda_1 = b_1 - c$, $\lambda_2 = b_2 - c$, $\lambda_3 = 2c$, $s = \frac{a_1 - 2c}{b_1 - c}$ and $v = \frac{a_2 - 2c}{b_2 - c}$.

4 Concluding Remarks

The use of representative agents in economic theory dates back at least as far as the late 1800s when Marshall's manuscript *Principles of Economics* saw the light of day.¹⁵ Marshall introduced the notion of a 'representative firm', but also considered employing this approach in other areas of economics.¹⁶ In fact, he is claimed to have said:¹⁷

“I think the notion of ‘representative firm’ is capable of extension to labour; and I have had some idea of introducing that into my discussion of standard rates of wages. But I don't feel sure I shall: and **I almost think I can say what I want to more simply in another way..**”

In this paper, we have shown this hunch might hold true for a well-known class of quadratic representative consumer models. Indeed, one can quite simply derive the corresponding linear oligopoly demand structure directly from a population of heterogeneous buyers. This renders the use of a fictitious agent in this case effectively redundant. Moreover, the resulting microeconomic foundation can be easily extended to other demand specifications.

It is, however, not only for the sake of simplicity that one should pass by this representative buyer model. In line with other recent work discussed above, we have pointed out some

¹⁵The first edition of this work was published in 1890. A flavour of the representative agent approach can also be found in Edgeworth (1925, an English translation of an Italian version from 1897).

¹⁶A detailed discussion is provided by Hartley (1996).

¹⁷See Pigou (1956, p. 437). Bold emphasis is ours.

problematic traits of this approach. In particular, we have argued that it is inaccurate as the representative agent's aggregate utility function has no clear connection with the represented buyers' objectives and that it is inadequate as it requires an unsatisfactory solution approach to obtain the linear oligopoly demand system. Moreover, it is quite possible that substitutability in prices embedded in the demand structure corresponds to products that the representative consumer considers complements. Together, this should raise strong doubts about welfare analyses based on this type of representative consumer models. It also naturally warrants critical assessment of other settings with a similar approach (*e.g.*, constant elasticity of substitution (CES) models).¹⁸ We leave this issue for future research.

¹⁸For this type of representative consumer model, see Dixit and Stiglitz (1977).

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